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Summary

Description of the two-phase cantilever rod experiment and benchmark setup

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WP4 – Flow-Induced Vibrations in multiphase flows

Deliverable D4.1 of the GO-VIKING project (101060826)
Multiphase cantilever rod experiment and benchmark setup

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Summary

The GO-VIKING project investigates FIV phenomena occurring in nuclear reactor under single- and two-phase flow conditions. The focus of the WP4 of the project is on the FIV phenomena induced by two-phase flows. The present document, as part of WP4, reports some experiments performed by University of Manchester. This experimental data will support numerical activities within GO-VIKING. The two-phase flow-induced vibration (FIV) of a free-clamped/clamped-free rod exposed to axial two-phase flow is experimentally investigated using a custom-made test piece which comprises a rod confined inside a tube. Both the vibrating rod and the confining tube are made of stainless steel: the rod is filled with lead, whereas the confining tube accommodates a transparent section for optical access. This provides a simple configuration with dimensions and linear mass density chosen to be informative of water-cooled nuclear reactor fuel rods. Following the WP2 activities in single phase flow, an air mixer chamber was introduced to generate the mixture (see Figure 1).

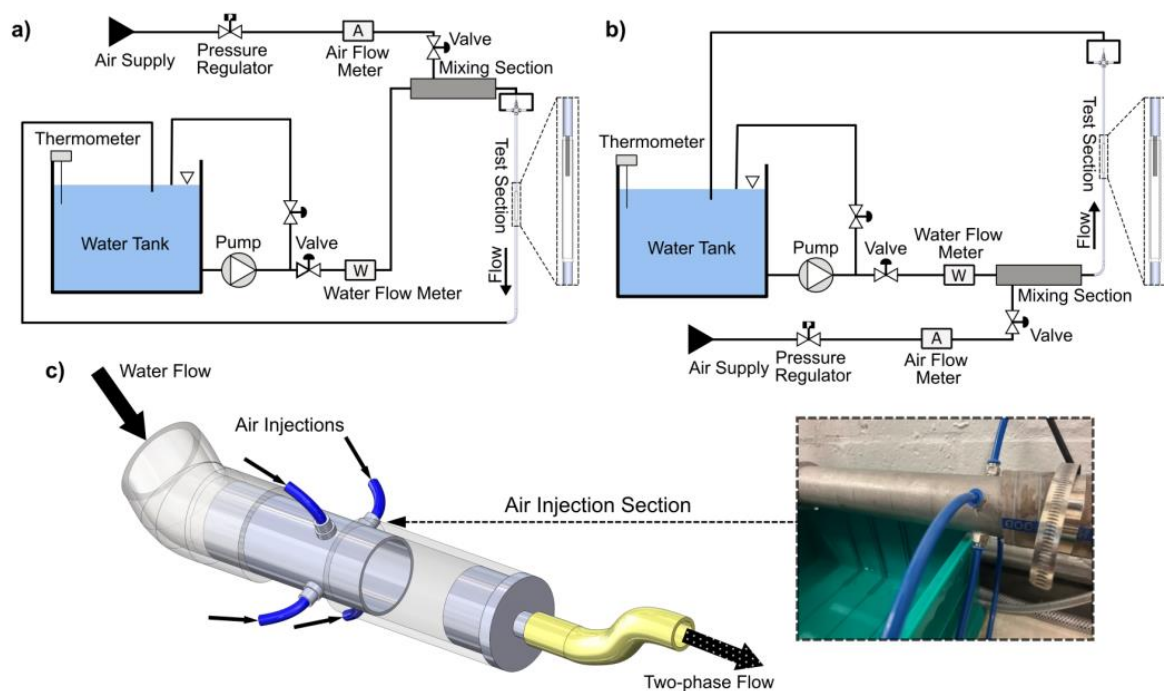


Figure 1 Test rig for the two-phase flow experiments: schematics for a) flow loop of the clamped-free configuration; b) flow loop of the free-clamped configuration; c) mixing section design with inset showing the real components for the air injection section.

Measurements are carried out using a non-contact technique where the motion of the rod is tracked based on a novel method that exploits the Hall effect. The air-water two-phase flow is characterized in terms of pattern based on fast video (see Fig 2). Experiments are carried out for different void fraction flowing towards the rod free-end (upward flow) and also while adopting the reverse configuration where the mixture flows away from the rod free-end (downward flow). During the experiments, the effects of the rod tip-end configuration, the flow Reynolds number, and the flow mixture were investigated.

D4.1: Multiphase cantilever rod experiment and benchmark setup

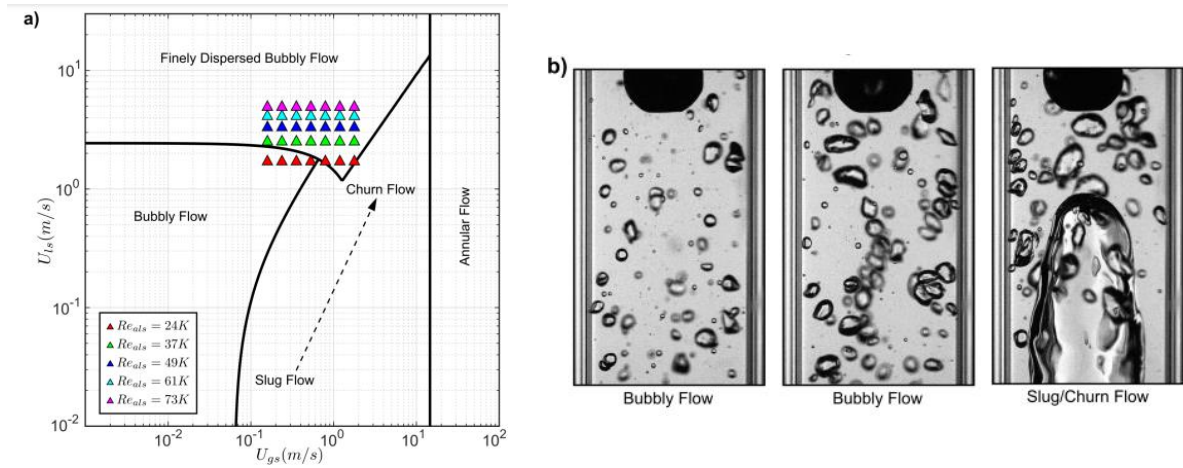


Figure 2 a) Adapted Taitel map for two-phase flow regimes and conducted experimental cases; b) example flow images for measurement cases at $Re_{annulus} = 24\,000$, void fraction increases from left to right.

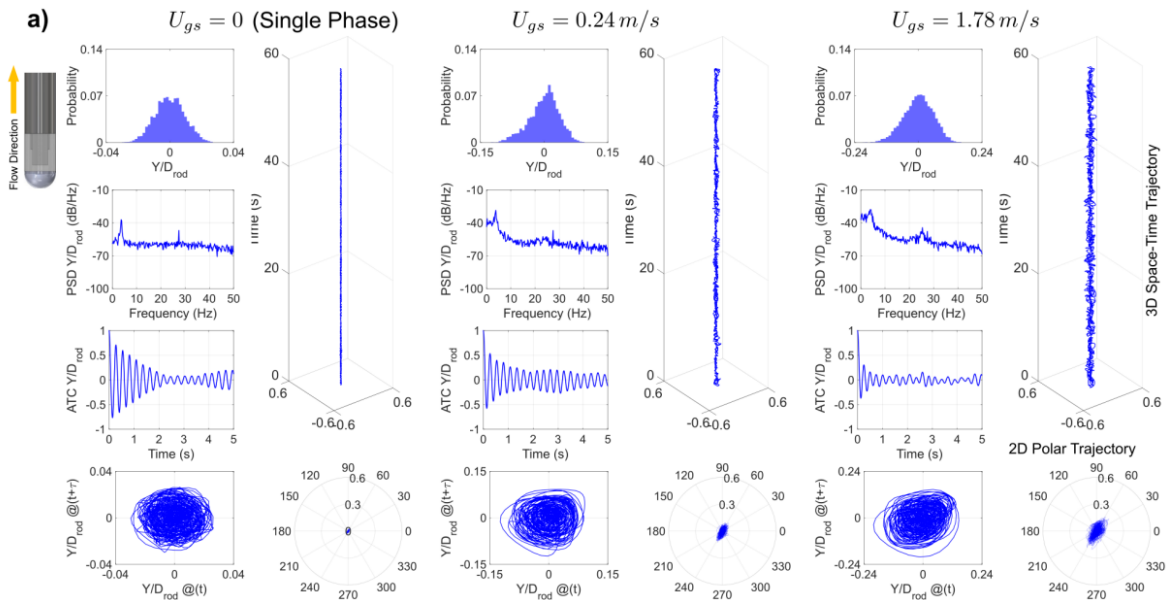


Figure 3 Example of rod vibration dynamics analysis for the hemispherical tip case in a free-clamped configuration at $Re_{annulus} = 24\,000$.

The successful identification of the rod vibration dynamics (see example in Fig 3) within two-phase flows builds upon a novel non-intrusive measurement technique developed in this study, overcoming the difficulty of conventional optical techniques when visual obscure degrades measurement accuracy. In fact, the results presented in this study represent the first comprehensive characterisation of rod vibration dynamics within axial two-phase flows.

In summary, the results for the rod vibration dynamics reveal a chaotic component alongside periodic oscillations. At low Reynolds numbers, the rod vibration dynamics features a dominant chaotic motion with small amplitude. As the Reynolds number increases, the periodic component gradually gains more importance. Particularly, for the rod with a cone tip shape, the rod vibration dynamics evolves into flutter-like oscillation at a Reynolds number of

49k under the single-phase water flow condition, hence a divergence in rod vibration amplitude is observed. Conversely, introducing two-phase flow is found to reduce the periodic component in rod vibration dynamics, and the effect increases with the increase in air-water void fraction. Two mechanisms are identified to contribute to such behaviour: an increased flow turbulence and unsteadiness due to the interaction between the air and water, and the impingement of air bubbles on the rod surface. Both mechanisms serve to increase random excitations experienced by the rod, leading to increased chaotic motion amplitudes. However, the latter effect also serves to reduce the fluid forces in conditions where movement-induced forces dominate flow excitations, e.g., the clamped-free hemispherical and cone tip cases at high Reynolds number, resulting in reduced periodic motion amplitudes.

Keywords

Axial flow; Vibration; Free-clamped rod; Experiment; Two-phase flow

Experiments on Axial-Flow-Induced Vibration of a Cantilever Rod in Two-phase Flow

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Flow-induced vibration (FIV) of a cantilever rod subjected to axial two-phase flow is experimentally investigated. The experimental setup features a stainless steel rod confined inside a vertical tube and clamped at the top end - a simple configuration yet informative of water-cooled nuclear fuel assemblies. The rod is filled with lead pellets to produce a linear mass density representative of nuclear fuel rods. Three end-pieces of the rod with blunt, hemispherical, and cone shapes are investigated. Experiments are conducted with flow passing the rod axially either from the top to the bottom in a “clamped-free” configuration, or from the bottom to the top in a “free-clamped” configuration. The flow involves two phases resulting from the mixture of air and water, with an air volumetric void fraction ranging between 0-0.5, covering a range of flow regime from single-phase to void-dominated flows. Water flow rate is varied within a range of superficial Reynolds numbers between 24-73 K. A novel non-intrusive methodology is developed to measure the rod vibration dynamics in axial two-phase flows, allowing the first comprehensive investigation of axial-FIV in such flow conditions. A consistent trend in rod vibration dynamics emerges across all investigated tip shapes, Reynolds numbers, and configurations: increasing the void fraction leads to increased chaotic motion amplitudes, but decreased periodic motion amplitudes. This is due to an increased flow turbulence resulting from air-water phase interactions, as well as increased random fluctuations in forces due to void impingement on the rod surface. At low Reynolds numbers, the increased random excitations lead to increased overall rod vibration amplitudes. However, at high Reynolds numbers, the reduced periodic force excitations can result in reduced overall motion amplitudes.

I. Introduction

Flow-induced vibration (FIV) of slender structures represents a paradigmatic problem in fluid-structure interactions, with applications in various engineering fields. Notable examples include aircraft wings and helicopter blades, ocean mining [1], energy harvesting [2–5], and nuclear power plants [6–9]. In recent years, the study of FIVs of slender cylinders exposed to axial flows, so-called axial-FIV, has received particular attention, driven by its applications in pressurized water-cooled nuclear reactors (PWR). In such reactor cores, axial flow is encountered by the nuclear fuel bundles, and the flow passes the fuel rods axially along the gaps between adjacent rods, leading to axial-FIVs that can result in long-term damage such as fretting wear and mechanical fatigue, which is a major contributor to fuel bundle failure [10, 11]. Therefore, understanding the mechanism of the fluid-structure interaction, involving slender cylinders and axial flows is essential for prolonging the lifespan of nuclear fuel bundles.

To date, studies on axial-FIV of slender cylinders have focused on single-phase flows, whereas axial-FIVs caused by gas-liquid two-phase flows, which are relevant for boiling water nuclear reactors, have been largely overlooked.

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Noteworthy studies on single-phase FIV include the work of Païdoussis [12], who developed a theoretical model describing the dynamics of slender cylinders under single-phase axial flows, where cantilevered cylinders for which flow moves axially from the clamped end to the free end, i.e., “clamped-free” configuration, were considered. The model successfully predicted the transition velocities for buckling and oscillatory instabilities for rod vibrations, which aligned well with his later experiments [13]. An increase in rod vibration frequency was found to be associated with the increase in flow velocity. The effect of various end-piece configurations on rod vibration dynamics was investigated in his experiments, showing that increasing the tapering of the end-pieces led to a reduction in critical flow velocities [13]. Rinaldi and Païdoussis [14] later extended the theoretical model to “free-clamped” cylinders, considering the configuration where the flow moves axially from the rod free end to the clamped end. In contrast to the clamped-free configuration, their experiments on the free-clamped rods with blunt and hemispherical end-piece shapes showed minor differences in rod dynamics behaviour. The rods initially exhibited small-amplitude oscillations at low flow velocities and subsequently transitioned to static divergence at high flow velocities, and the process was accompanied by a decrease in rod vibration frequency [14].

Flow interaction mechanisms for axial-FIV have also been investigated from early on, with a focus on single-phase flows. It is well-known that below the divergence flow velocities for which the rod vibration develops from small random motions to large amplitude oscillations, the flow interaction mechanism primarily arises from random pressure fluctuations within the turbulent boundary layer along the rod’s surface [9, 15]. These fluctuations create an unsteady transverse force that primarily affects the rod at its fundamental natural frequency [1]. The significance of turbulence excitations was confirmed by the numerical study of Salachna et al. [9], where different turbulence models were employed to simulate axial-FIV of cantilevered rods. Their results showed that compared to the eddy-viscosity model, the Reynolds stress model, which in complex flows returns lower and more realistic turbulence levels, provides a more accurate prediction for turbulence excitations, which is essential for producing rod vibrations observed in experiments. Cioncolini et al. [7] conducted experiments on axial-FIV of cantilevered rods, considering free-clamped configuration with blunt and hemispherical end-piece shapes, where both rod vibration dynamics and flow fields were recorded using optical tracking and particle image velocimetry (PIV). Their results highlighted the effect of “movement-induced excitations” at high flow Reynolds numbers, arising from the transverse flow velocity component as the rod vibration displacement increases. In their experiments, a larger vibration amplitude was found for the blunt tip shape compared to the hemispherical tip shape, due to more pronounced movement-induced excitations. Cioncolini et al. [8] later extended their experiments to both clamped-free and free-clamped cantilevered rods, focusing on a hemispherical end-piece shape. Their results showed that for clamped-free rods, unsteady separation near the rod free end adds an additional source of excitation at high flow Reynolds numbers. The effect of tip shape on axial-FIV of cantilever rods was also recently investigated by Li et al. [16]. Their experiments involved blunt and cone shaped end-pieces with different height-to-diameter ratios, considering both clamped-free and free-clamped configurations. Their results showed that the tip shape of the cantilevered rod serves to modulate the effect of unsteady flow separation, and that rods with a cone tip shape resulted in larger vibration amplitude in the clamped-free configuration, but a slightly reduced amplitude in the free-clamped configuration, due to enhanced/reduced vortex-wall interactions.

For air-water two-phase flows in vertical tubes, some early work exists that characterises the flow behaviour for different ratios of air and water phases. The ratio can be defined using the homogeneous void fraction [17]:

$$\alpha = \frac{U_{gs}}{U_{gs} + U_{ls}} \quad (1)$$

where U_{gs} and U_{ls} are the superficial velocities (defined in Eq. 2) for the air and water phases, respectively. As is well-known, the homogeneous void fraction provides a good approximation to the actual void fraction for two-phase flows with negligible slip between the phases, such as the bubbly flows of interest here. Hewitt and Hall-Taylor [18] proposed four basic flow patterns as void fraction increases for upward air-water two phase flows: bubbly flow, slug flow, churn flow, and annular flow. These flow patterns were mapped based on the superficial velocities of the air and water phases by Taitel [17] for two-phase flows inside a vertical tube; and similarly by Ulbrich and Mewes [19] for flows in tube bundles. Currently, there is no dedicated study on axial-FIV of cantilevered rods in two-phase flows, leaving a notable gap in the understanding of the physical mechanisms involved. The lack of dedicated experiments has resulted in the absence of datasets for the validation of numerical methodologies. That said, experimental investigation of axial-FIV in two-phase flows poses significant challenges to the current measurement techniques. The challenges arise due to the wide range of observed flow behaviour at different air volumetric void fractions and the optical interference of the air bubbles in various flow regimes. Due to such challenges, classical optical methods based on image processing fail to provide an accurate account for the flow characteristics and rod vibration dynamics. In the current study, we

present a novel non-intrusive technique developed for the measurement of the rod vibration dynamics in two-phase flows, overcoming the difficulty of conventional optical methods in bubbly flow environments. Experiments are conducted for a simple cantilevered rod configuration subjected to axial air-water two-phase flows. The investigation covers a comprehensive range of flow conditions representative of water-cooled nuclear fuel assemblies. The results presented in the current study represent the first comprehensive characterisation of axial-FIV of a cantilevered rod in two-phase flows reported, covering a wide range of rod dynamics behaviour from single-phase to void dominated flow regime.

II. Methods

A. Experimental Setup

The experiments in the current study are conducted using a simple open-loop setup consisting of a water tank with flow loop driven by a centrifugal pump (10HP Leeson pump). The setup is an upgraded version of our previous study [16], with additional flow paths to allow air injection for air-water two-phase flow experiments. Within the flow loop, a test section is installed which hosts a rod in the centre of a confining tube at an upright vertical position. The rod is clamped at the top end and free at the bottom. Flow is allowed to pass through the annulus between the rod and the confining tube either from the top to the bottom, resulting in an clamped-free configuration, or from the bottom to the top, leading to a free-clamped configuration. Schematics of the experimental rig and flow loop for both the clamped-free and the free-clamped configurations are provided in Fig. 1a-b. A direct immersion digital thermometer (ETI-Electronic Temperature Measurements Ltd; temperature range: 248–343 K; accuracy ± 1 K) is used to measure the temperature of water allowing for estimation of its density and viscosity. The mass flow rate of water is measured using a turbine flow meter (Cole-Parmer; measurement range 37.8–378 L/min; accuracy ± 3.78 L/min) installed upstream of the test section. The air injection system is connected to the main flow loop via a mixing section, Fig. 1c. Within the mixing section, four injection ports are located around the air injection pipe, which are arranged in an circumferential manner. The injection ports are controlled by individual valves to allow better mixing of air and water upstream of the test section. For the experiments conducted in this study, air supply is provided via a compressed source at a constant supply pressure of 4 Bar. A flow meter (SMC PFM750S, measurement range 0–50 L/min; accuracy ± 0.5 L/min) is used to measure the total flow rate for air injections. The superficial velocities for air and water, U_{gs} and U_{ls} , are obtained using:

$$U_{gs} = \frac{Q_{gs}}{A}; \quad U_{ls} = \frac{Q_{ls}}{A} \quad (2)$$

where Q_{gs} and Q_{ls} are the volumetric flow rates for air and water measured using the flow meters; A is the internal cross sectional area of the confining tube. For convenience, we define the superficial annulus Reynolds number, Re_{als} , based on the water phase:

$$Re_{als} = \frac{\rho U_{als} D_{hyd}}{\mu} \quad (3)$$

where ρ is the water density; μ is the water dynamic viscosity. The hydraulic diameter, D_{hyd} , is taken as the annular gap between the confining tube and the rod ($D_{hyd} = D_{tube} - D_{rod}$), and the velocity, U_{als} , is taken as the annulus superficial velocity of the water phase (obtained by replacing A with the annular cross section area in Eq. 2 for the water phase). Experiments are conducted for five different Reynolds numbers, within a range between 24–73 K. For each Reynolds number, eight different air injection flow rates are considered, with the corresponding U_{gs} varying between 0–1.78 m/s. Note that, a single-phase water flow case with $U_{gs} = 0$ is also included. The experiment cases investigated in the current study are summarised in Table 1, including the range of void fraction, α , covered in each Reynolds number case. Additionally, Fig. 2a shows the experiment cases against the Taitel map [17] adapted for the current experiment conditions, illustrating the flow regimes for the different cases. It is evident that most of the experimentally investigated cases, over the higher Reynolds number range, fall into the bubbly flow regime, whereas slug/churn flow is only encountered for the lowest range of Reynolds numbers investigated. Representative two-phase flows for the current experiments are also illustrated in Fig. 2b, where an example hemispherical rod tip at a Reynolds number of 24 k is used for this demonstration.

Details of the test section including dimensions of the cantilevered rod are shown in Fig. 3a. A union cross (Swagelok SS-25M0-4) is used to clamp the rod at the top end while at the same time leaving two horizontal ports for the flow loop connection. The confining tube is made of AISI 316 stainless steel which has an inner/outer diameter of 20.86/25.06 mm. The test section is rigidly fixed to a ground frame throughout the experiments. In the region near the rod tip, the confining tube is replaced with a transparent Perspex section with the same inner/outer diameter to allow

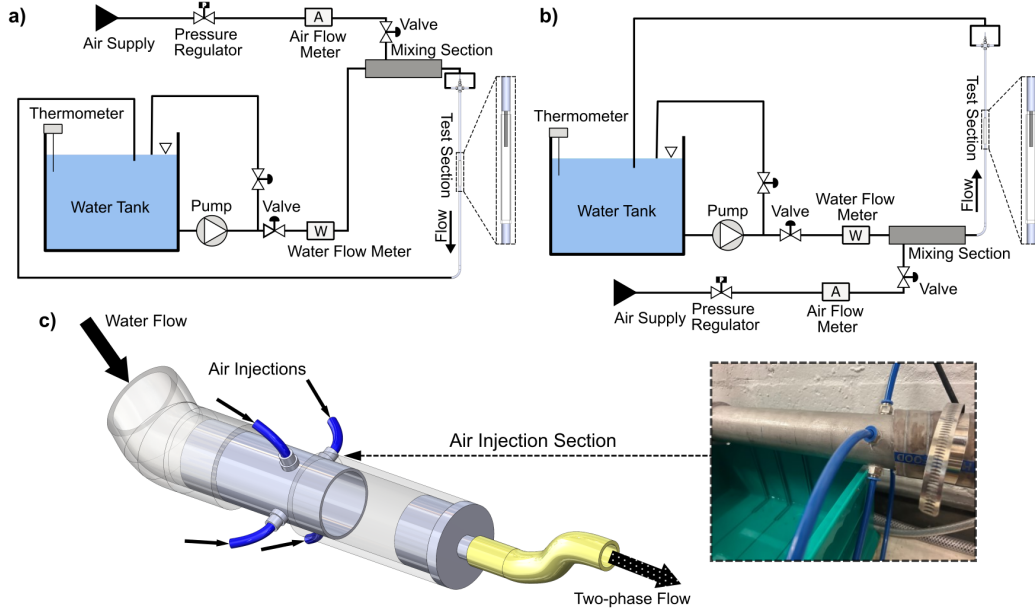


Fig. 1 Test rig for the two-phase flow experiments: schematics for a) flow loop of the clamped-free configuration; b) flow loop of the free-clamped configuration; c) mixing section design with inset showing the real components for the air injection section.

Table 1 Flow variable range for axial two-phase FIV

Re_{als}	$U_{ls}(m/s)$	$U_{gs}(m/s)$	α
24 K	1.7	0-1.78	0-0.51
37 K	2.5	0-1.78	0-0.42
49 K	3.3	0-1.78	0-0.35
61 K	4.1	0-1.78	0-0.30
73 K	4.9	0-1.78	0-0.27

optical access. In order to remove any optical distortions due to the cylindrical surface, a transparent Perspex box with square cross section and flat surfaces is used to host the transparent section, Fig. 3a. During the experiments, the gap between the box and the tube was filled with water to eliminate any lensing effect for optical measurements. A flow straightener is installed at the bottom of the test section which eliminates swirl and secondary flows before entering the straight tube. The flow straightener consists of three mildly squashed pipes of 100 mm in length inserted into the confining tube, Fig. 3a. For the free-clamped configuration, the upstream flow recovery distance, extending from the top of the flow straightener to the free end of the rod is approximately 50 times the diameter of the confining pipe (1090 mm for the blunt tip case, Fig. 3a), which is sufficient to enable achieving fully-developed flow conditions. This has been validated by measuring the flow velocity profiles upstream of the rod tip for a range of Reynolds numbers. For more detailed discussions on these validation experiments, the reader can refer to our previous studies [8, 16]. For the clamped-free configuration, the flow going into the test section results from the merging of the two symmetric streams entering from the horizontal ports of the union cross. This setup serves to minimise localized cross flows near the rod clamped end by joining the two opposite flow velocities. Note that, for this configuration, the primary flow disturbances arise from the mixing of the two streams near the rod clamped end, whereas the flow disturbance before entering the union cross have relatively small impact. Therefore, no flow straightener is installed in the feeding pipes.

The rod utilized in this study is a circular tube made of AISI 316 stainless steel with an inner/outer diameter of 8.83/10.01 mm. The tube is filled with lead shot of loose texture to mimic the mechanical loading of the fuel pellets in nuclear fuel rods. The lead shot has a density of 11.34 g/cm^3 and size range between 0.3-1.6 mm, which is densely filled and evenly distributed inside the rod tube with a packing ratio of 84.6% to ensure that no motion of the shots inside

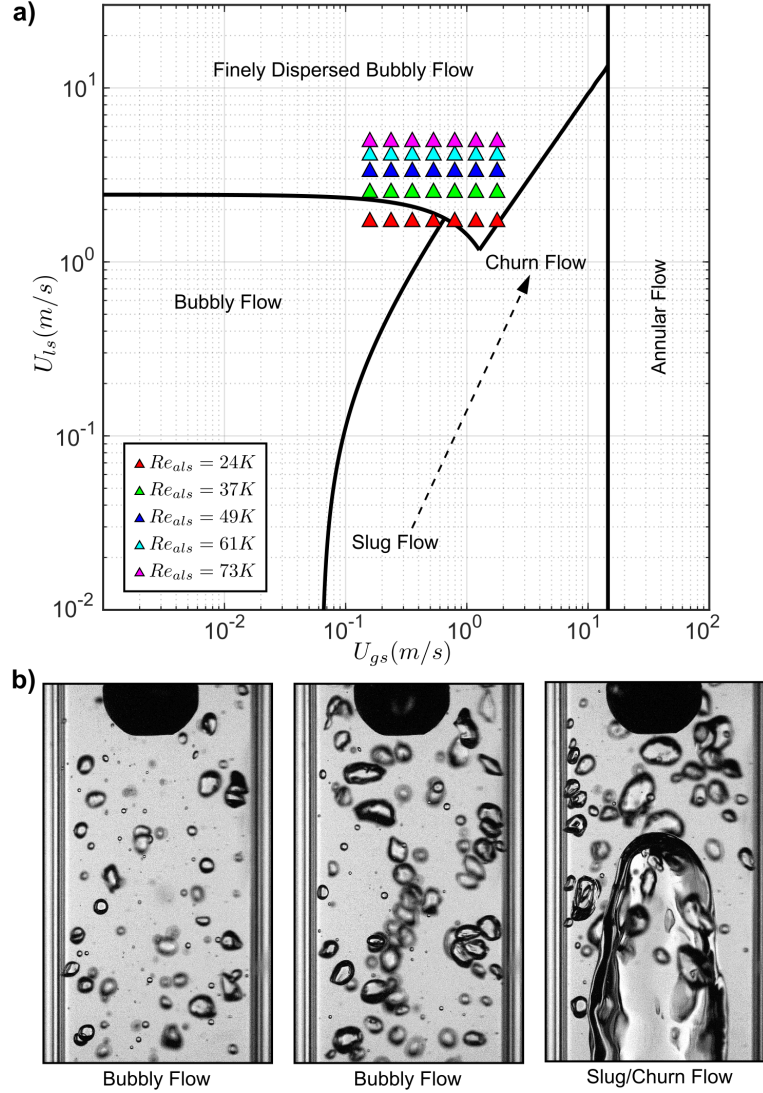


Fig. 2 a) Adapted Taitel map for two-phase flow regimes and conducted experimental cases; b) example flow images for measurement cases at $Re_{als} = 24K$, void fraction increases from left to right.

the tube exists during the experiments. Before running any experiments, a homogeneity test for the rod was conducted by measuring the rod centre of mass, which showed that lead filling resulted in less than 0.5% shift in the centre of mass for the rod. Hence, the rod is considered to have constant linear mass density along the rod axis. The resulting linear mass density of the rod, accounting for both the tube material and the lead shot filling, is 5.88 g/cm . On the other hand, due to its loose texture the lead shot filling has no influence on the rod flexural rigidity. An initial free vibration experiment was conducted to confirm the rod flexural rigidity, and the reader can refer to our previous work for detailed discussions on these verification experiments [8].

Figure 3b shows the schematics of the design for the different rod tips investigated in the current study. The rod tube is sealed with an aluminium cap with a mass of approximately 2.1 g. A cylindrical sensor mount-piece is attached to the flat end surface of the aluminium cap using secure adhesive. This component is used for the non-intrusive method developed for measuring the rod vibration dynamics, which will be introduced in the next section. The sensor mount-piece has flat surfaces, which act as the end-piece for the blunt tip case investigated in the current study. For the cone and hemispherical tip cases, an additional end-piece is attached to the lower flat surface of the sensor mount-piece. Both the sensor mount-piece and the end-pieces are 3D printed using light-weight hard resin, hence the influence of their mass on rod vibrations are deemed negligible.

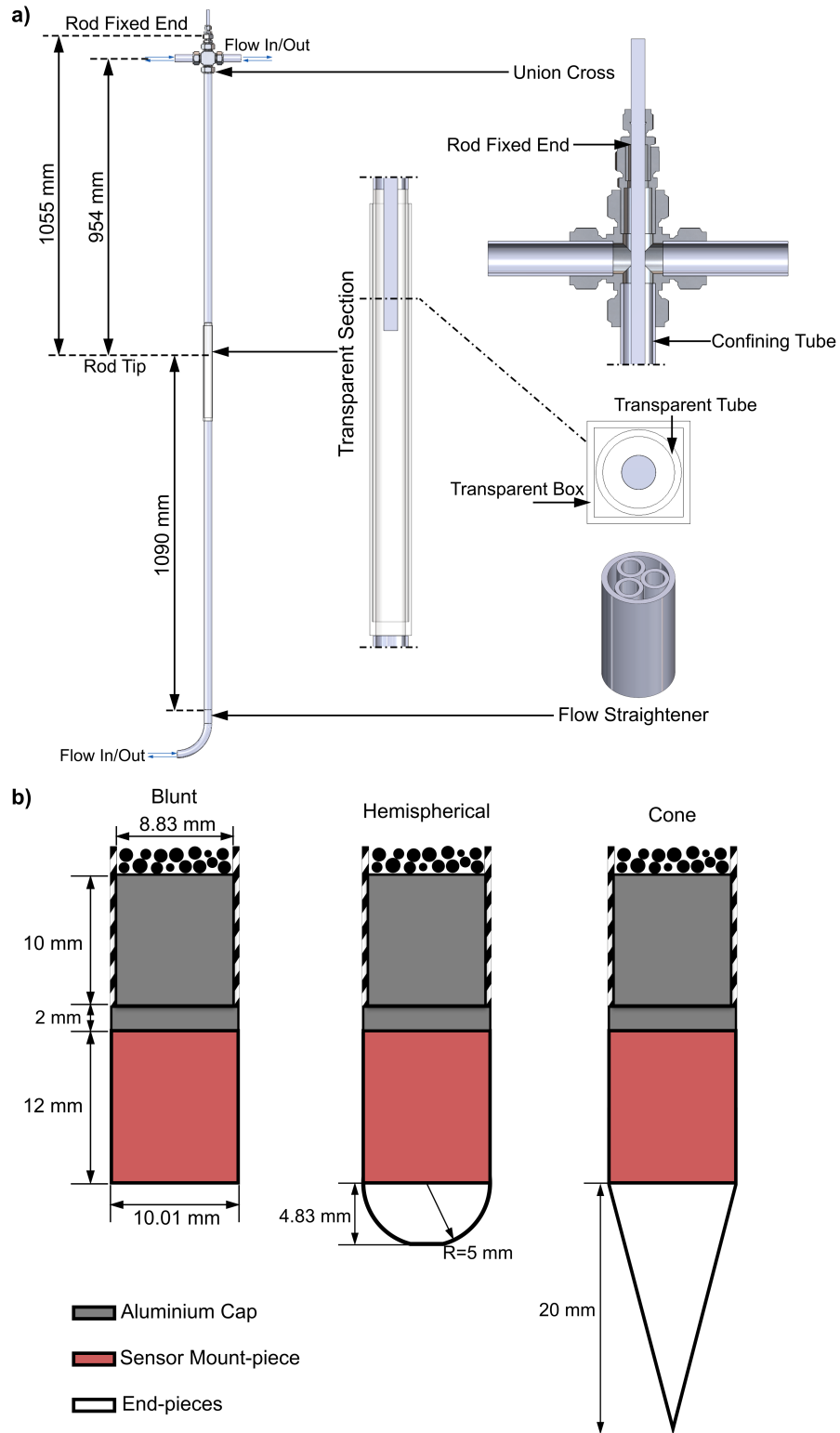


Fig. 3 a) Schematics of the test section components. Rod and tube lengths for the blunt end rod setup are provided; b) schematics and dimensions of rod tip design for different tip shapes.

B. Hall-Effect Measurement Method

1. Measurement toolkit development

A simple non-intrusive method for the measurement of the rod vibration under two-phase axial flows has been developed in this study, based on the Hall effect sensors. Specifically, the methodology exploits Hall effect sensors to measure the position of the vibrating rod tip, and hence to track its motion during axial-FIVs. The method overcomes the difficulty in classical optical-based non-intrusive techniques when air-water two-phase flow is considered, where air bubbles tend to block the rod area leading to degradation in image processing accuracy. The main components of the developed toolkit are schematically shown in Fig. 4a, where the sensor mount-piece is of cylindrical shape with internal slots holding permanent magnets (RS PRO Neodymium Disc Magnet, Diameter 4 mm) inside. The sensor external block also features four slots that allow attachment of the hall-effect sensors (Honeywell SS495A Ratiometric Linear Sensor). The sensors are placed in the middle of the block edges, covering four sides of the measurement section aligning with the sensor mount-piece, allowing homogeneity in sensitivity of measurement when the rod moves in different directions. During the experiments, the sensor mount-piece is directly attached to the free end of the rod, Fig. 3b, whereas the sensor external block is attached to the transparent box at the same vertical position. A schematic of the installed sensor components on the experimental rig is shown in Fig. 4b. The Sensor output voltages were acquired using a data acquisition board (NI USB-6210, National Instruments) at a sampling rate of 1 kHz. Calibration was conducted by recording the sensor output values for all the sensors when the rod was placed at different locations within a Cartesian grid. This process together with the coordinate system definition is schematically shown in Fig. 4c. The origin of the coordinate system is at the centre of the square block area. A total of 120 calibration points were recorded, with the centroid of the cross-section of the sensor mount-piece varying within a range between -7 and 7 mm in both coordinate directions. Note that, the maximum rod displacement within the confining tube is less than 6 mm, hence the calibration data covers the entire range for possible rod displacement in the current experiments. For each measurement point, 6000 sensor readings for a duration of 60 seconds were recorded for each sensor, and the average value was used to create the calibration matrix.

During axial-FIV experiments, rod vibration displacement for a duration of 60 seconds were recorded and used for the subsequent analyses of vibrational dynamics. The raw measurement data was processed using a 6-order Butterworth low-pass filter to eliminate high frequency noise above 50 Hz, a value that is well-above the rod's natural frequency for the current experiment setup ($f_0 = 3.7\text{Hz}$). The filtered data was used to deduce the time series of rod tip position using the calibration matrix based on a biharmonic spline interpolation method within Matlab. The obtained rod tip displacement time-series were then used for the subsequent analyses including the displacement power spectrum, the autocorrelation function, and reconstruction of the attractor in phase-space. For computing the power spectrum, the Welch method [20] was employed, and the error in the dominant frequencies was determined using half of the full width at half maximum (HWHM) of the corresponding peak in the power spectrum. Rod displacement amplitude was determined using the root-mean-square (RMS) value of the time series data. For the developed method, several factors can contribute to the measurement error, including the vibrational noise pickup from the environment, noise generated during the data acquisition process such as the uncertainties in the sensor output values and the data processing/interpolating procedures. To evaluate the measurement error for the RMS rod displacement, preliminary measurements were made when the pump was switched off and the cantilevered rod remained motionless in stagnant water. In such a condition, as the rod remain motionless, the only contributor to the measured rod displacement time series is the aforementioned errors during the measurement process. Hence, the RMS values of the fluctuating rod displacement time series during such tests were used to estimate the overall measurement error for the RMS rod displacement. Such experiments were repeated several times, yielding an error of less than $40\text{ }\mu\text{m}$. An example result of such tests, including the time series data, displacement distributions, power spectra (PSD) and autocorrelation functions (ATC) is shown in Fig. 5a-b. The distributions for both front and side displacements follow a Gaussian profile; the power spectra is primarily broadband; and the autocorrelation functions quickly decay with no obvious periodicity, indicating the measured displacements are the random noise pickup of the system.

2. Validation

The developed method was validated against the well-established optical technique for rod vibration measurement. The baseline optical method employed for this validation was developed for the study of axial-FIV of cantilevered rods in single-phase flows, which has been introduced in our previous studies [7, 8, 16], hence, will be described briefly here. The setup consists of a high-speed imaging camera (Sony Alpha A6300; lens: Sony E 16-50mm f/3.5-5.6) that

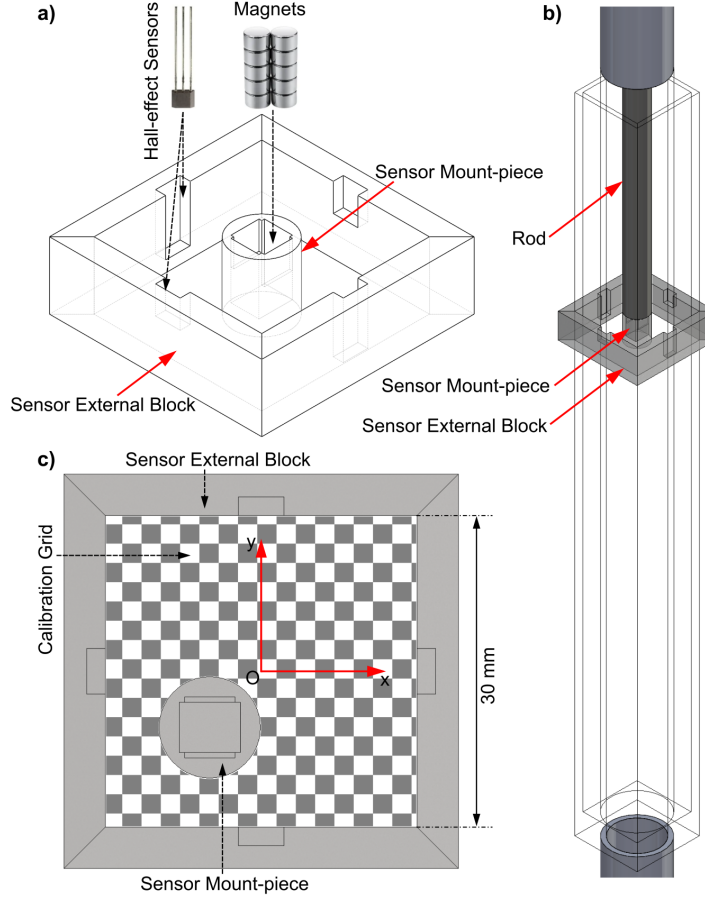


Fig. 4 a) Schematics of the main components of the measurement toolkit for two-phase FIV; b) schematic of the installed measurement toolkit for the current experimental setup; c) schematics of sensor calibration method and coordinates.

is placed in front of the transparent test section, recording the rod motions at a frequency of 100 frames per second. A mirror was placed at a 45-degree angle on the side of the transparent section to capture the rod motions on the perpendicular plane. LED panels are placed on the back and side of the transparent section for illumination. The optical setup for the validation experiments is shown schematically in Fig. 6a. Figure 6b shows an example image of the camera recording for a test case, where the interrogation windows for the front and side displacements are marked out. Rod displacement is determined using an image processing algorithm developed by our group for axial-FIV and fluid structure interaction studies [3–5, 21–23], where the position of the centroid of the rod area within the interrogation windows for each image frame is computed to produce the displacement time-series. For the validation experiments, rod vibrations are simultaneously measured using the developed measurement toolkit. The setup for the toolkit can be seen clearly within the the captured image in Fig. 6b. Both the optical method and the measurement toolkit produces time-series displacements for both the front and side motions of the rod tip. The time-series data are then processed to form the validation datasets.

For the validation experiments, the cone shaped rod tip configuration is employed, see Fig. 3b. Experiments are conducted using single-phase water flows for both clamped-free and free-clamped configurations, at four specified Reynolds numbers within a range of 24-91 K. The comparison of the resulting radial RMS displacements (R_{rms}) and dominant rod vibration frequencies of the rod motion within the front plane (f_x) using the developed toolkit and the optical measurement method at different Reynolds numbers are shown in Fig. 7a. Additionally, the deviations in the measured values using the developed toolkit compared to those obtained using the optical method are provided in Table 2. The results show an excellent agreement between the measured rod displacements and frequencies using the two different approaches. For the rod RMS displacement values, a higher deviation value is found for lower Reynolds

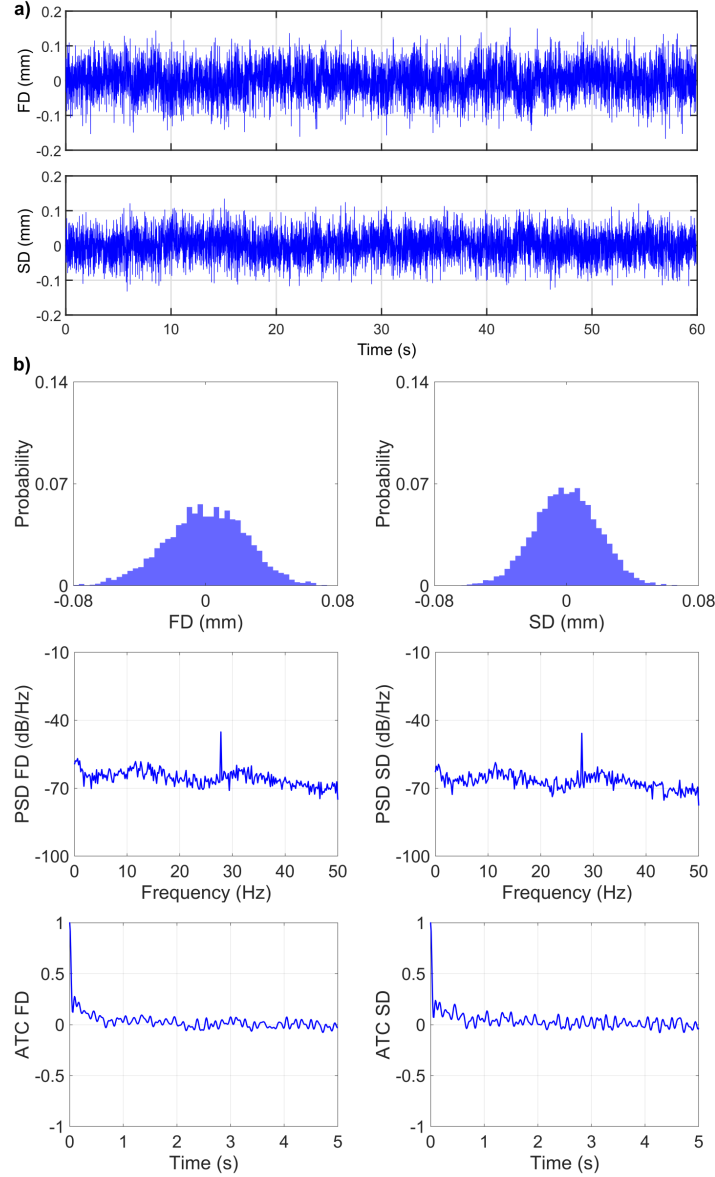


Fig. 5 a) Displacement time series data for rod in stagnant water; b) displacement distributions, power spectra, and autocorrelation functions for the data series. FD/SD indicates front/side displacement.

number cases, which is expected, due to the larger relative uncertainties in the measurements for these flow conditions. However, the measured rod vibration frequencies match almost exactly in all considered cases. The measured rod vibration dynamics using both methods are further exemplified in Fig. 7b for a case at Reynolds number of 65 K, in a clamped-free configuration, where the rod tip trajectory, the power spectra extracted from the time series data, and the reconstructed attractor in phase-space using the delay embedding method [24] are presented. The rod vibrations show a dominant periodic oscillation mode in the perpendicular plane, where a limit cycle can be observed from the phase plot using both measurement approaches. Peaks in power spectra agree well between the methods near the dominant mode frequency, despite some slight differences in higher peak energy emerge. The minor differences in higher frequency peak energies and tip trajectories might be due to a slight misalignment in the coordinate axes between the developed method and optical measurement setups.

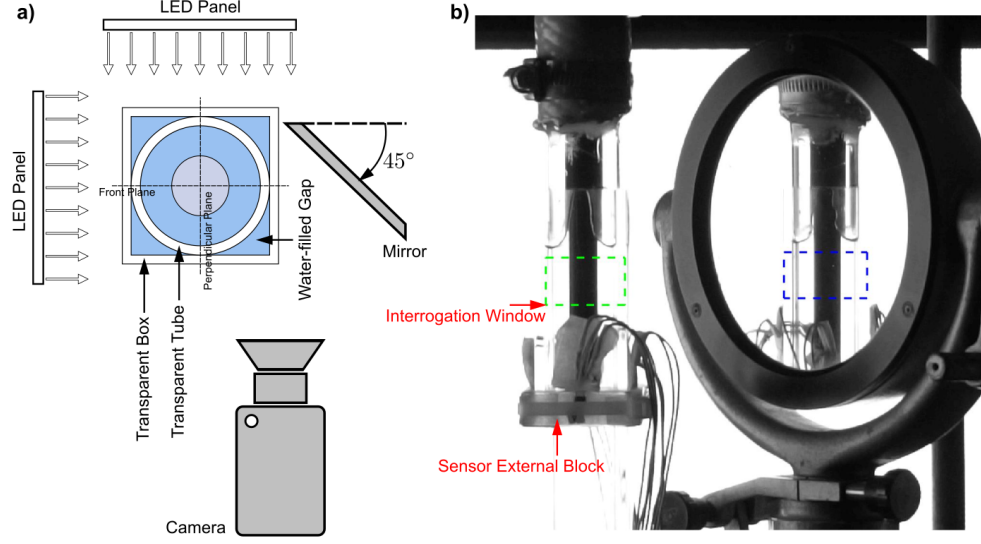


Fig. 6 a) Experimental setup for optical measurement of rod vibrations; b) example of captured image frame for displacement processing.

Table 2 Deviations in measured rod RMS displacements and frequencies

Configuration	Reynolds number (K)	RMS Disp. Deviation	Frequency Deviation
Clamped-free	24.5	-36%	-1%
	41.0	22%	0%
	65.1	-1%	0%
	89.4	-3%	1%
	25.3	23%	2%
Free-clamped	42.2	5%	0%
	66.7	-1%	0%
	90.8	8%	0%

III. Results and Discussions

A. General Trends of Rod Vibrations in Two-phase Flows

1. Effects of Reynolds number

The variation of the rod vibration amplitude and frequency with the Reynolds number (Re_{als}) is firstly presented. Fig. 8 shows the radial RMS amplitude of the rod and its vibration frequency for the clamped-free configuration. At each Reynolds number, five cases with different superficial air velocities are included in this demonstration, with the case with $U_{gs} = 0$ corresponding to the single-phase water flow condition. Each column in Fig. 8 corresponds to a different rod tip shape, see Fig. 3. Illustrations of the rod tip shapes as well as the flow directions are included in the inset figures. Starting with the single-phase water flow cases, shown with red diamond markers in Fig. 8, both the blunt and hemispherical tip shape cases show an approximately linear increase in vibration amplitude with the increase in Reynolds number (on the log-log scale). The hemispherical tip shape case generally shows higher vibration amplitude than the blunt tip case, and ultimately hits the tube wall at a Reynolds number of 73k. For the cone tip shape case, the rod vibration amplitude is initially small and increases slowly with the increase in Reynolds number, however, at Reynolds number of 49k, the amplitude suddenly surges to a large value and the rod hits the tube wall for all higher Reynolds numbers values. The two-phase flow cases show that the RMS amplitude of oscillations generically increase with the value of the superficial air velocity, U_{gs} . For all the tip shapes, a larger superficial air velocity leads to a

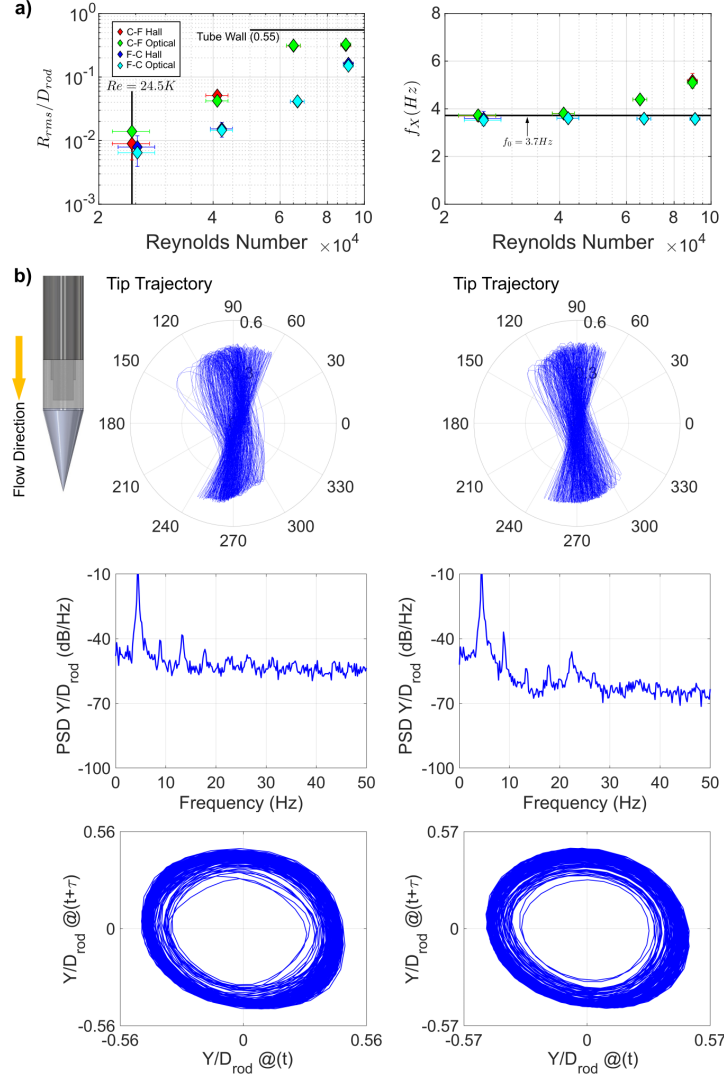


Fig. 7 a) Comparison of measured rod RMS displacements and frequencies using the developed toolkit and the optical technique; b) example of a measured rod vibration dynamics for the clamped-free configuration at Reynolds number of 65 K using both methods. C-F/F-C indicates clamped-free/Free-clamped configuration; Hall/Optical indicates results obtained using the developed method/optical measurement; in b), left/right column corresponds to results obtained using the developed method/optical technique.

larger rod vibration amplitude at low Reynolds numbers, however, with the increase in superficial air velocity, the slope of increment in amplitude becomes smaller. In fact, for the highest superficial air velocity case, $U_{gs} = 1.78 m/s$, a decreasing trend in rod vibration amplitude is observed. Notably, the divergence in rod vibration amplitude for the cone tip shape case, observed in the single-phase water flow condition, is absent within two-phase flow conditions. In terms of the rod vibration frequency, a similar trend is observed for all cases including both single-phase and two-phase flows. The frequencies gradually increase with the increase in Reynolds number, with minimum values hinged at the rod's fundamental natural frequency, $f_0 = 3.7 Hz$, at the lowest Reynolds number of $24k$. However, a slight positive shift in frequencies is observed for the blunt tip case at the highest superficial air velocity.

The corresponding effects of two-phase flow on the free-clamped, upward flow, configuration are presented in Fig. 9. An increasing trend in rod vibration amplitude with the increase in Reynolds number is found for all tip shape cases and across all superficial air velocities. Similar to the clamped-free configuration, the slope of amplitude increment with the increase in Reynolds number becomes smaller at higher superficial air velocities. It is also noted that, under two-phase

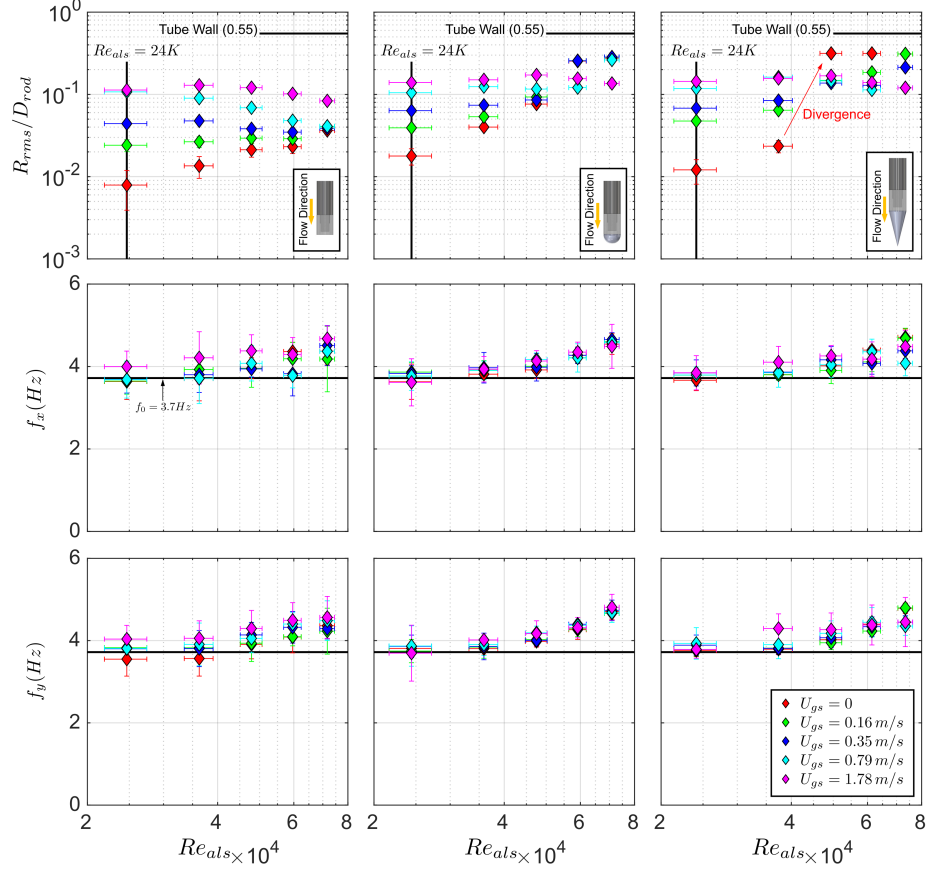


Fig. 8 Variation of rod vibration amplitude and frequency with Reynolds number for the clamped-free configuration. Insets illustrate the corresponding tip shape and flow direction for each column results.

flow conditions, the effect of changing the Reynolds number on rod vibrations is less pronounced, particularly when the Reynolds number is small. This can be seen in Fig. 9, where the rod vibration amplitude at low Reynolds number values is independent of the Reynolds number. The rod vibration frequencies are found to decrease with the increase in Reynolds number for the blunt and hemispherical tip shape cases, and the trend is consistent across all the superficial air velocities. However, the blunt tip cases show a more significant decrement in frequency with Reynolds number. On the other hand, rod vibration frequencies for the cone tip cases appear to be insensitive to changes in Reynolds number. This observed behaviour is in agreement with previous studies on axial-FIV [7, 8, 16].

2. Effects of void fraction

The effects of void fraction on rod vibration dynamics is presented in this section. Fig. 10 shows the effect of the void fraction on the rod vibration amplitude and frequency for the clamped-free configuration, for five different Reynolds numbers, Re_{als} , from 24k to 73k.. At low Reynolds numbers ($Re_{als} = 24 - 37K$), a consistent increasing trend in rod vibration amplitude with the void fraction is observed for all tip shape cases; however, when the Reynolds number becomes higher, the increment slope becomes weaker. At the higher end of the Reynolds number range examined ($Re_{als} = 61 - 73K$), the trend for the blunt tip case differs from that of the hemispherical and cone tip cases, where the former shows an increasing trend but the latter two show a decreasing trend in vibration amplitude. For the hemispherical and cone tip cases, the decreasing amplitude at higher Reynolds numbers and the increasing amplitude at lower Reynolds numbers converge at approximately $\alpha = 0.2$, which is marked on the figures. Notably, for all the tip shape cases, the effect of changing void fraction on the vibration amplitude is more significant at lower void fraction values. Nevertheless, as the void fraction becomes greater than 0.2, the increment/decrement in rod vibration amplitude becomes less noticeable, and the amplitude tends to a similar value for all Reynolds number cases. On the other hand,

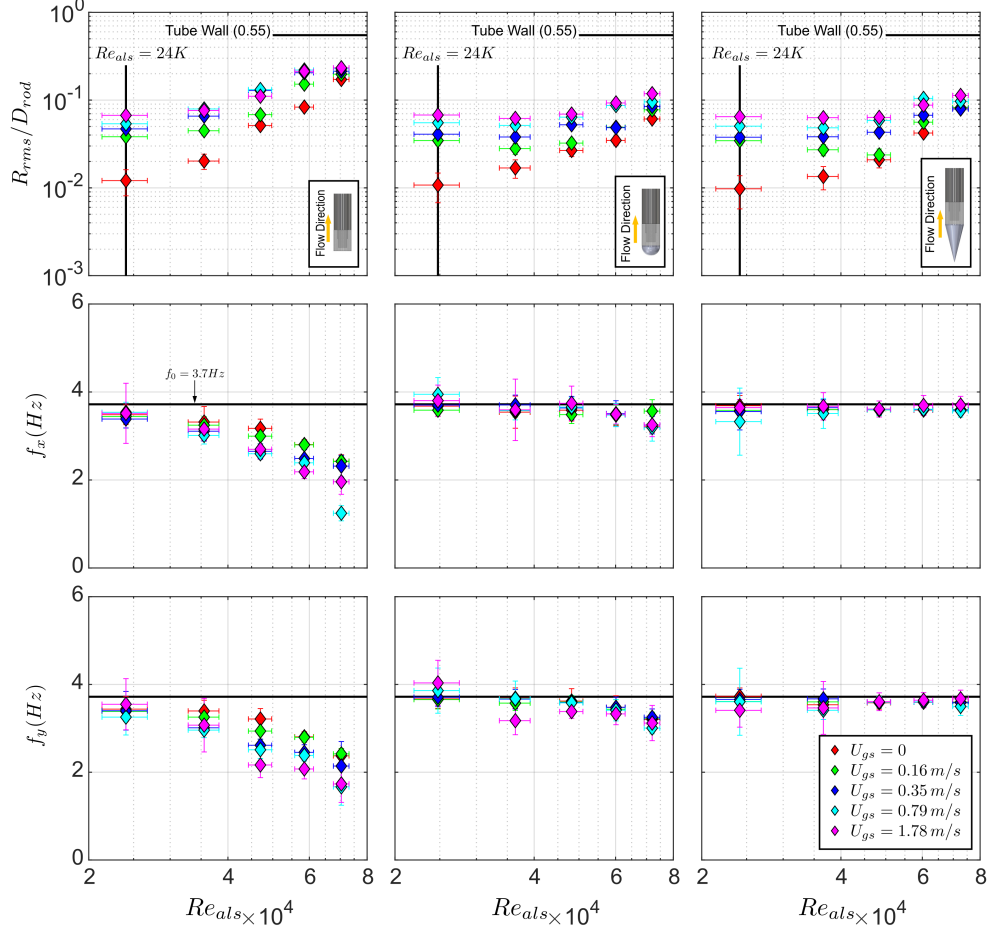


Fig. 9 Variation of rod vibration amplitude and frequency with Reynolds number for the free-clamped configuration. Insets illustrate the corresponding tip shape and flow direction for each column results.

there appears to be no discernible pattern in the rod vibration frequencies as the void fraction changes. However, for a specific Reynolds number, the variation in rod vibration frequencies tends to be small.

Figure 11 shows the variation of rod vibration amplitude and frequency with the void fraction for the free-clamped, upward flow, configuration. Similar to the clamped-free configuration, a more pronounced increase in the rod vibration amplitude with void fraction is observed for lower Reynolds number cases and at low void fraction values. The variation trend, however, is consistent across the different tip shapes. Once again, we mark the $\alpha = 0.2$ line on the amplitude figures. It is noticed that, when the void fraction increases above 0.2, the change in rod vibration amplitude becomes small for all Reynolds number cases. Subtle decreases in rod vibration frequencies with the increase in void fraction is observed for the blunt tip cases at relatively high Reynolds numbers. The frequencies variation with void fraction, however, is generally small.

B. Rod Vibration Dynamics and Flow Interaction Mechanisms

In this section we take a closer look at the vibration dynamics involved. For each experimental case, several diagrams are produced from the measured time series data to assess the dynamic characteristics of the rod. These include: the normalized histograms of the displacement time-series, the power spectrum, the autocorrelation function, and the reconstructed attractor for the phase-space. Results are presented for two distinct Reynolds numbers of 24k and 73k, representing the lower and upper limits for the current range of investigation. Note that, the two cases cover the pre- and post-divergence conditions for the cone tip shape case, Fig. 8. For each Reynolds number, results at three distinct flow conditions are presented, including the single-phase water flow case, serving as baseline reference; a two-phase flow

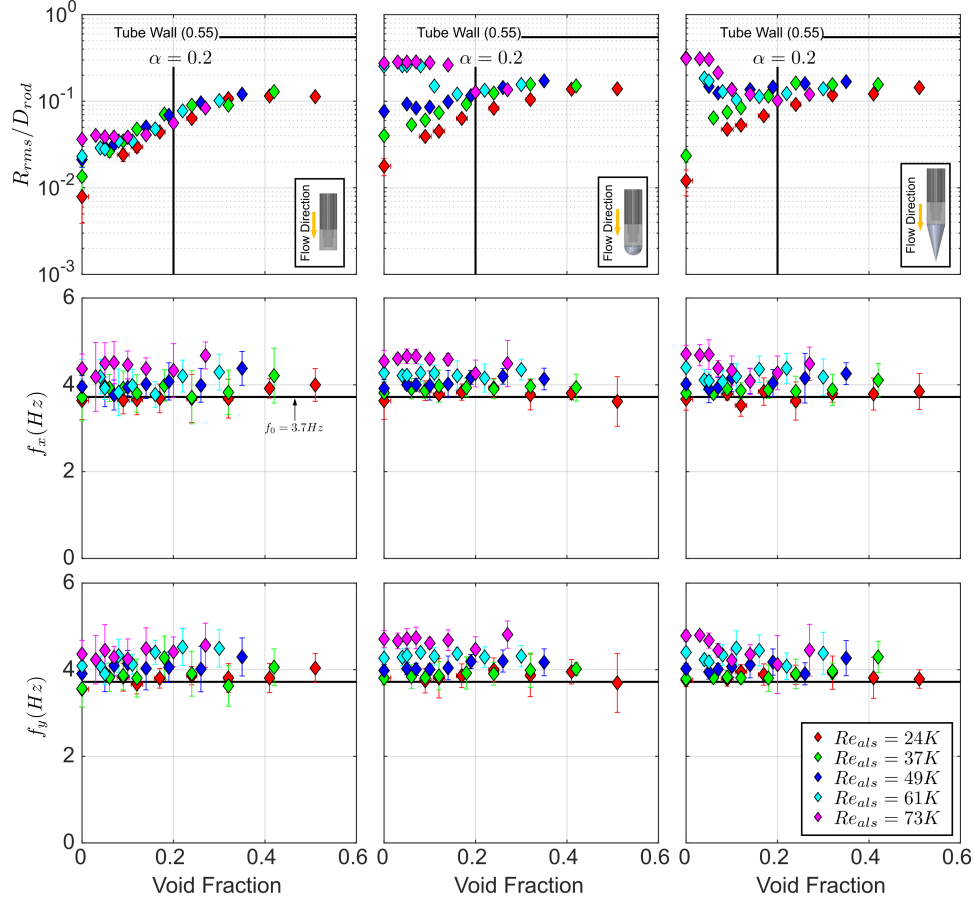


Fig. 10 Variation of rod vibration amplitude and frequency with void fraction for the clamped-free configuration. Insets illustrate the corresponding tip shape and flow direction for each column results.

case with void fraction of less than 0.2, where the rod vibration dynamics is more sensitive to variations in Reynolds number and void fraction (see Fig. 10); and a two-phase flow case with void fraction greater than 0.2, where the rod vibration amplitude becomes insensitive to changes in flow conditions. Figures 12-13 present the dynamics diagrams for the clamped-free configuration, along with the 3D space-time illustration and 2D polar map for the rod tip trajectory. The blunt and cone tip shape cases are chosen for this demonstration, representing the two extreme cases of taperness of the rod tip. Note that, for the presented cases, the dynamic characteristics for the front and side rod motions (i.e., displacements within the front and perpendicular planes, see Fig. 6a) are qualitatively similar, hence only the diagrams for the rod motion within the perpendicular plane are presented.

The dynamics of a cantilevered rod under axial flows typically consists of a chaotic component along with periodicity. This is well-established in several previous works involving axial-FIVs of cantilevered rods within single-phase flows [7, 8, 16]. For the two-phase flow results shown here, the dynamics characteristics depicted in Fig. 12-13 show qualitatively similar behaviour to single-phase flow cases. Specifically, for both tip shapes presented, at low Reynolds number (Fig. 12a and 13a), for both single-phase and two-phase flow cases, the rod motion primarily consists of small random vibrations at the rod's fundamental natural frequency. In this case, the displacement histogram follows roughly a Gaussian profile, the autocorrelation functions exhibits periodicity but are fast-decaying, and the reconstructed attractors are "blob-like". The rod dynamics is referred to as fuzzy period-1 motion [25]. At high Reynolds number (Fig. 12b and 13b), the dynamics behaviour for the blunt tip shape show minor differences to that of the low Reynolds number case, however, for the cone tip shape, the results for the single-phase and two-phase flow cases show different features. For the single-phase flow case, the displacement histograms become flattened, the decaying of the autocorrelation functions becomes slower, and the reconstructed attractors tend to be roughly "ring-shaped", indicating that the rod

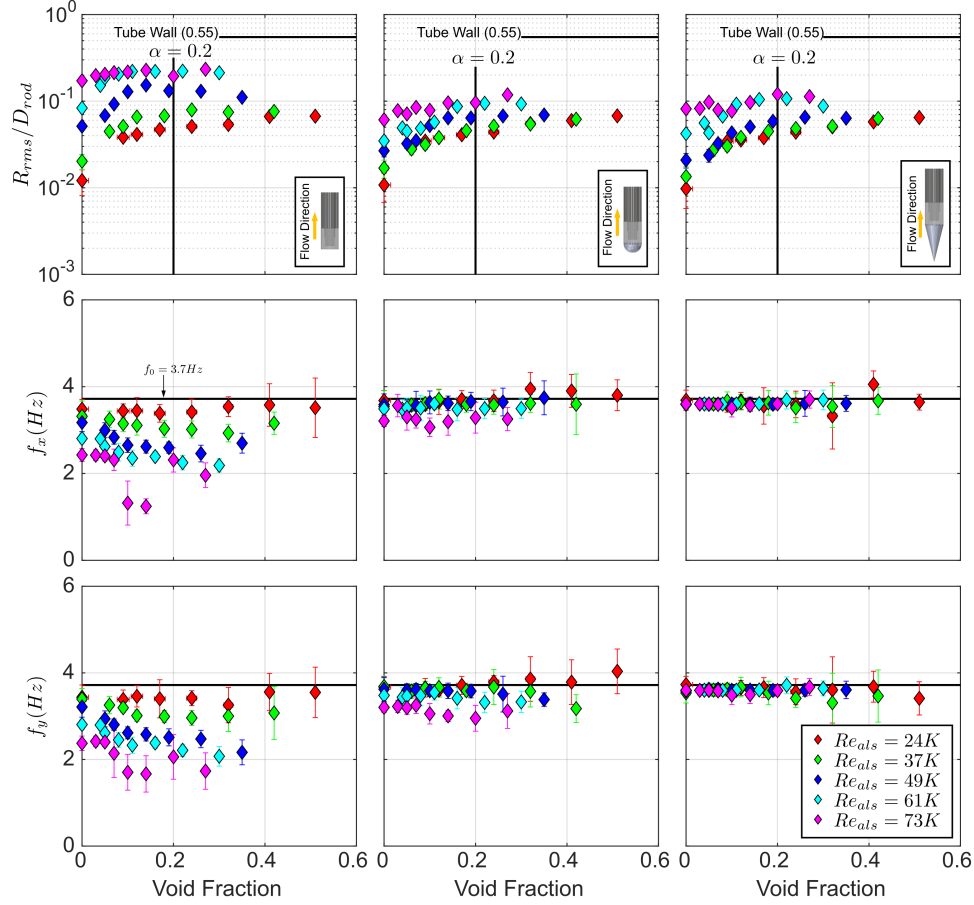


Fig. 11 Variation of rod vibration amplitude and frequency with void fraction for the free-clamped configuration. Insets illustrate the corresponding tip shape and flow direction for each column results.

motion has evolved into a flutter-like large-amplitude oscillation after the divergence Reynolds number of 49k; whereas, for the two-phase flow cases, the rod vibration dynamics remain dominated by the chaotic component, despite a larger periodicity can be observed for the $U_{gs} = 0.24 \text{ m/s}$ case. The different dynamics behaviour between the blunt and cone tip shape cases in the single-phase flow condition can be explained by the mechanism of unsteady flow separation, where the cone tip shape case with a larger taperness near the rod tip experiences less flow separations, leading to a larger periodic fluid force excitations as the rod displacement builds up at higher Reynolds numbers, hence a larger vibration amplitude and ultimately the flutter divergence, as shown in Fig. 8 and 13b [16].

Comparing the single-phase flow case with the two-phase flow cases, for the blunt tip rod, at both Reynolds numbers, the rod dynamics show consistently fuzzy-periodic behaviour, despite a larger rod vibration amplitude is clearly observable for the two-phase flow cases from the 3D space-time and 2D polar map of the rod tip trajectory. This indicates that the primary flow excitation for rod vibration remains similarly of random nature in two-phase flow conditions, and that the amplitude of the excitations increases with the increase of void fraction. This observation suggests that for axial-FIV in two-phase flows, the interactions between air and water leads to increased turbulence and unsteadiness of the flow, which is the main source of excitation for small amplitude chaotic rod vibrations [1, 15]. Furthermore, the effect is enhanced as the void fraction increases, leading to larger random excitations, hence larger chaotic motion amplitudes of the rod. Additionally, we note here that the enhanced random excitation in two-phase flows is also a result of the impingement of the air bubbles on the rod's surface. This effect is clearly more pronounced for relatively high void fraction conditions. Such impingement would lead to random fluctuating forces acting on the rod's surface due to the difference between the density of air and water, which ultimately leads to increased random excitations for rod vibrations. On the other hand, for the cone case, at low Reynolds numbers, the rod shows similar

dynamics behaviour to that of the blunt tip cases, i.e., fuzzy-periodic with a vibration amplitude that increases with the increase in void fraction. However, it is also observed that with the increase in void fraction, the autocorrelation function for rod vibrations decays more rapidly, indicating a reduced periodic component in the rod motion. Such reduction in periodicity in vibration dynamics is more evident for the high Reynolds number case, where the rod vibration switches from a flutter-like oscillation to a fuzzy-periodic motion, associated with a reduction in vibration amplitudes. The transition in dynamics behaviour indicates that the rod experiences a reduced periodic force excitation. Such reduction in periodic force excitation is clearly due to the impingement of the low density air bubbles on the rod's surface, which reduces the motion-induced forces acting on the rod's surface for relatively large displacements.

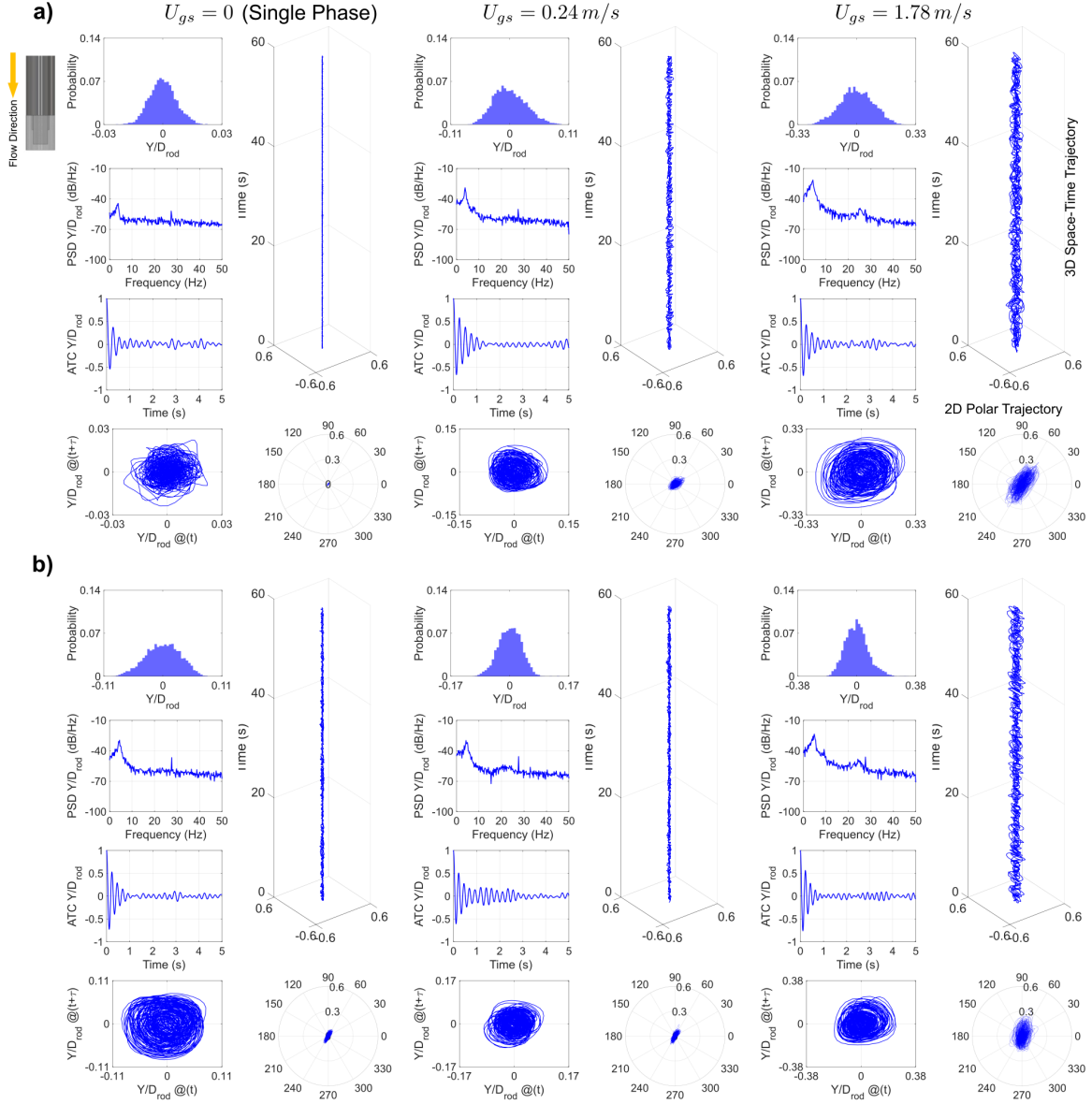


Fig. 12 Example of rod vibration dynamics analysis for the blunt tip case in a clamped-free configuration at a) Reynolds number of 24k and b) Reynolds number of 73k, including histograms of displacement time series, PSD, autocorrelation function, and phase-space topology.

Finally, the rod vibration dynamics for the free-clamped configuration are presented in Fig. 14. For this configuration, the rod dynamics behaviour is qualitatively similar among the tip shape cases investigated, hence, only the hemispherical tip shape results are presented, as shown in Fig. 14. For this configuration, the rod vibration dynamics is fuzzy-periodic

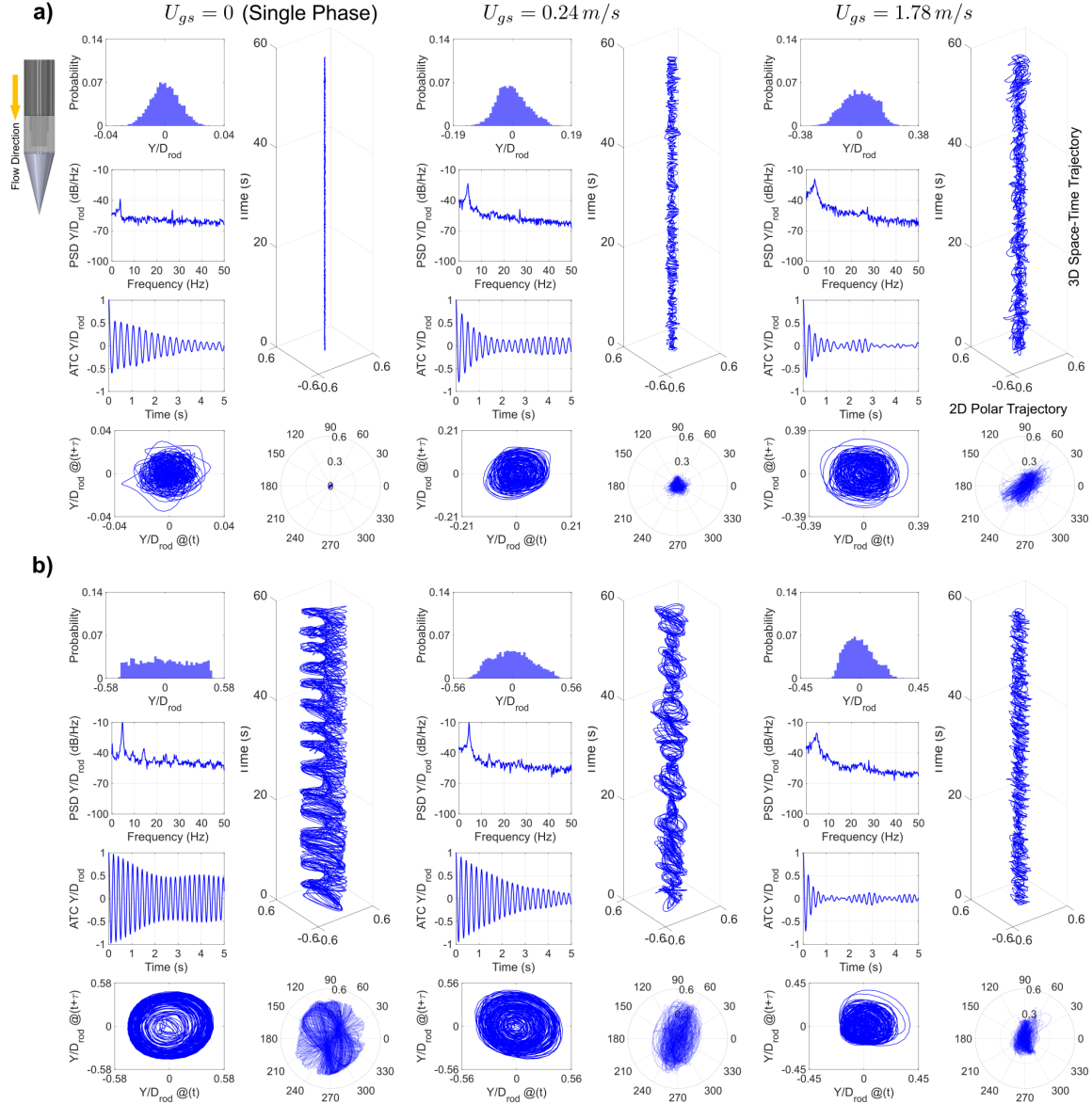


Fig. 13 Example of rod vibration dynamics analysis for the cone tip case in a clamped-free configuration at a) Reynolds number of 24k and b) Reynolds number of 73k, including histograms of displacement time series, PSD, autocorrelation function, and phase-space topology.

for all the Reynolds number and superficial air velocity cases considered. Similar to the clamped-free configuration, with the increase of void fraction, the periodic component of rod vibration tends to decrease at both Reynolds numbers. This is identified by the more rapid decaying of the autocorrelation function. However, despite the reduced periodicity in rod vibrations, the rod vibration amplitude increases with the increase in void fraction, due to the increased random excitations of the flow.

IV. Conclusions

Experiments have been conducted for flow induced vibrations of a cantilevered rod within axial two-phase flows consisting of a mixture of air and water. The study involves a comprehensive range of flow conditions, with the Reynolds number defined using the water phase lying in a range that is informative of water-cooled nuclear power plants, and

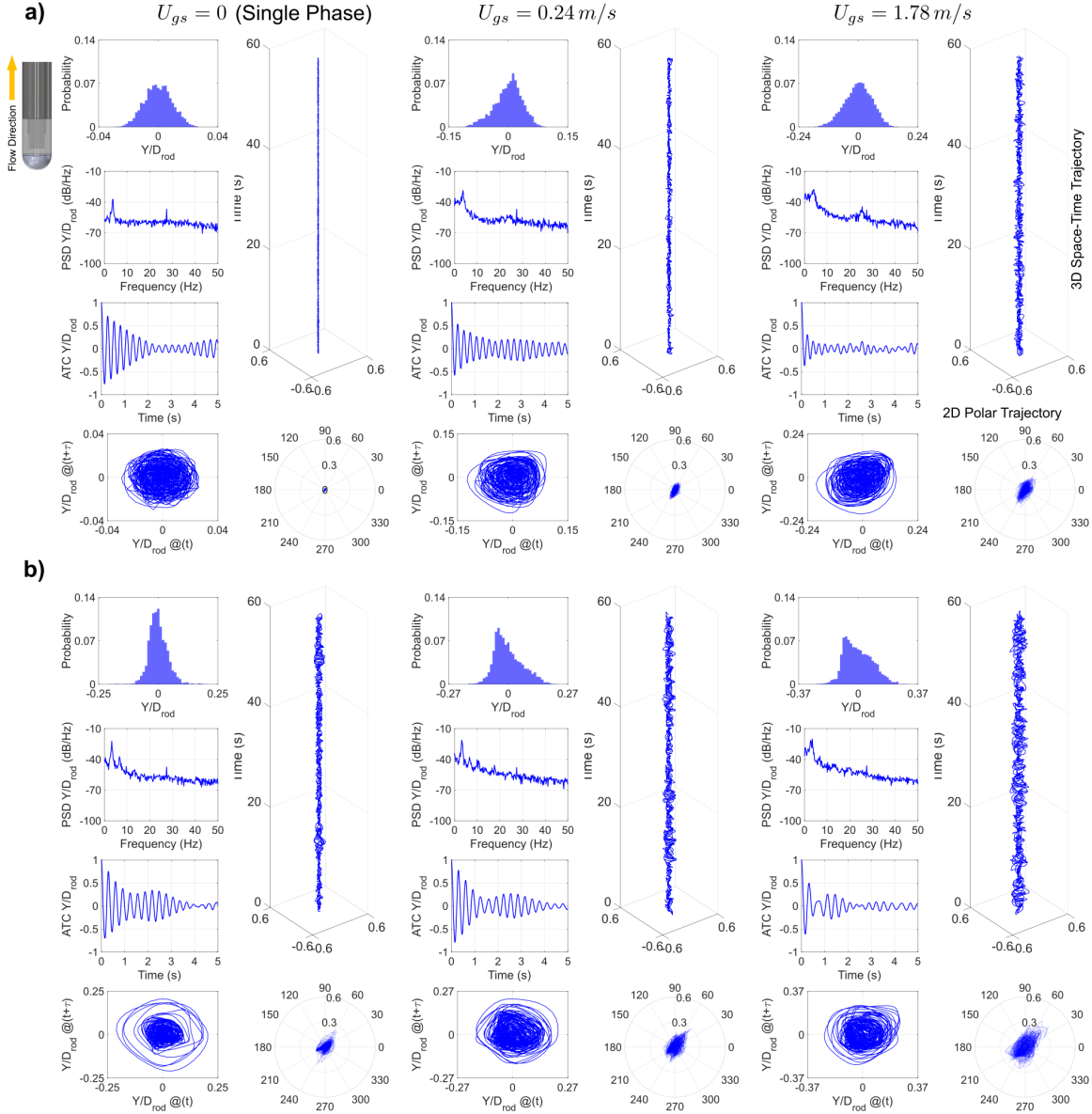


Fig. 14 Example of rod vibration dynamics analysis for the hemispherical tip case in a free-clamped configuration at a) Reynolds number of 24k and b) Reynolds number of 73k, including histograms of displacement time series, PSD, autocorrelation function, and phase-space topology.

the air-water volumetric void fraction in a range covering single-phase to void dominated flow regime. Measurements are conducted for a simple configuration, featuring a single cantilevered rod confined in a circular tube, where the flow moves either from the root to the tip of the rod, featuring a “clamped-free configuration”, or from the tip to the root of the rod, referred to as the “free-clamped configuration”. The successful identification of the rod vibration dynamics within two-phase flows builds upon a novel non-intrusive measurement technique developed in this study, overcoming the difficulty of conventional optical techniques when visual obscure degrades measurement accuracy. In fact, the results presented in this study represent the first comprehensive characterisation of rod vibration dynamics within axial two-phase flows. The rod vibration dynamics reveal a chaotic component alongside periodic oscillations. At low Reynolds numbers, the rod vibration dynamics features a dominant chaotic motion with small amplitude. As the Reynolds number increases, the periodic component gradually gains more importance. Particularly, for the rod with a cone tip shape, the rod vibration dynamics evolves into flutter-like oscillation at a Reynolds number of 49k

under the single-phase water flow condition, hence a divergence in rod vibration amplitude is observed. Conversely, introducing two-phase flow is found to reduce the periodic component in rod vibration dynamics, and the effect increases with the increase in air-water void fraction. Two mechanisms are identified to contribute to such behaviour: an increased flow turbulence and unsteadiness due to the interaction between the air and water, and the impingement of air bubbles on the rod surface. Both mechanism serves to increase random excitations experienced by the rod, leading to increased chaotic motion amplitudes. However, the latter effect also serves to reduce the fluid forces in conditions where movement-induced forces dominate flow excitations, e.g., the clamped-free hemispherical and cone tip cases at high Reynolds number, resulting in reduced periodic motion amplitudes.

Acknowledgments

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